Small Growth and Distress Anomalies: Two Sides of the Same Coin?

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Abstract

We develop a firm valuation model with growth options and downward jump risk as a proxy for distress to capture the cross-sectional variation of stock returns associated with high distress, size and book-to-market. A greater jump risk reduces the ratio of fixed assets to firm value due to a greater option value (Merton (1976)) and simultaneously decreases the sensitivity of firm value to systematic risk. We propose a novel mechanism that simultaneously generates the low equity returns of small firms with low book-to-market ratio (small growth anomaly), and the low equity returns of high distress firms (distress anomaly) shown to exist separately in the literature. The model further predicts that the stocks of high distress firms capture the risk of small growth firms; and that this relation strengthens in the firms' reliance on growth options. Empirical results support these predictions.

Keywords: Distress, size, book-to-market, small, growth, glamor, stocks, failure risk, default risk, anomalies, cross section of stock returns, asset pricing, real options, growth options, mixed jump-diffusion process.

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1 Introduction

The literature highlights the existence of cross-sectional patterns in stock returns related to size and book-to-market ratio (Fama and French (1993)). For example, the stocks of small firms earn higher average returns (the size effect hereafter), as do the stocks of high book-to-market firms (the book-to-market effect hereafter). Some deviations in patterns exist, however. Fama and French (1996) document that the stocks of small firms with low book-to-market ratio tend to have negative abnormal returns (small growth anomaly hereafter). In a different strand of the literature, Dichev (1998) and Campbell, Hilscher, and Szilagyi (2008) document that firms with high default risk also tend to have abysmally low stock returns (distress anomaly hereafter), casting doubt on the notion that the size or the book-to-market effect captures compensation for distress risk.[1] In this paper, we reconcile these seemingly disparate empirical regularities in a firm valuation model with growth options and negative jump risk.[2] The model formalizes the main ideas and predictions which are then verified empirically.

In the model, firms can expand a finite number of times in response to demand shocks by irreversibly investing in physical capital. At any instance, firms face the possibility of distress proxied by a non-systematic risk of encountering a negative jump in asset value. The possibility of distress is decreasing in firm maturity, and firm maturity is proxied by the number of previous expansions. Similarly to Carlson, Fisher, and Giammarino (2004), operating leverage results with per period fixed operating charges that increase in capital stock. When demand for a firm's product decreases, equity value falls relative to book value, proxied by the capital stock, and the riskiness of returns increases due to greater operating leverage generating the book-to-market effect. Firm size, on the other hand,

[1]Distress is commonly invoked to explain the size (Chan and Chen (1991)) and the book-to-market (Fama and French (1996); Vassalou and Xing (2004)) effects. The idea is that the stocks of distressed companies tend to move together, so their risk cannot be diversified away and investors require a premium for bearing such risk.

[2]We model distress risk in reduced-form by allowing the possibility of a sudden loss in asset value following the approach used in numerous intensity-based models in the credit risk literature (Duffie and Singleton (2003)).

captures the importance of growth options relative to assets-in-place contributing to the size effect. While revenue betas are assumed constant, firm betas are nonetheless time-varying and reflect past expansions as well as current demand.

In addition to the separate size and book-to-market effects, we arrive at a new economic role for distress in explaining expected returns. Every thing else equal, a greater jump risk increases the value of growth options and raises the ratio of firm value to capital stock. Since jump risk is non-systematic, the greater option valuation reduces the systematic proportion of total firm value and lowers the firms' beta. As a consequence, the model simultaneously generates lower expected equity returns and lower book-to-market ratios for high distress firms. Based on this result, the model rationalizes what empirically appears to be persistent market overvaluation of younger, smaller and highly distressed firms (Conrad, Kapadia, and Xing (2012)), and reconciles the small growth and the distress anomalies evidenced empirically.

The main mechanism driving this result hinges on a little-known and counterintuitive property of option pricing first formally shown by Merton (Merton (1976)). Everything else the same, increasing a stock's non-systematic risk of abruptly taking a downward jump should be accompanied by a commensurate increase in the drift of the price process in order for the market-determined expected rate of return on the stock to remain constant. This translates to a greater drift in the risk-neutral measure which is a condition for a greater call option value on the stock. In a similar vein, incorporating growth options on high jump risk assets produces a direct linkage between low expected returns, high firm valuation ratios, and high distress in line with the evidence in the cross-section of firms.

The bulk of our empirical analysis is focused on verifying this link. We first show through descriptive analysis that the highest distress decile firms based on O-Scores (Ohlson (1980); Griffin and Lemmon (2002);Dichev (1998)) share similar firm-characteristics as the smallest and the lowest book-to-market quintile firms. We show that these groups of firms have abysmally low average stock returns; are among the most distressed based on O-Scores; are the smallest based on market capitalization; have growth, instead of value characteristics; are among the youngest; and have the worst credit rating in the cross-section. We also show that these groups of firms have below average financial leverage, highlighting that poor operating performance, rather than financial distress, is the likely culprit for these firms' high O-Scores. This feature of the data motivates our own growth option-based explanation for the distress anomaly.

In order to further verify the cross-sectional relation between high distress and small growth, we examine whether the returns of a zero-cost investment strategy composed of the most distressed stocks can explain the returns of a zero-cost investment strategy composed of the smallest and the lowest book-to-market stocks. We show that the high distress strategy explains, and completely subsumes, the negative average returns of the small growth strategy while controlling for the Fama and French 3 factors. The results confirm that high distress firms contain important information that captures the risk of small firms with low book-to-market in line with the model's predictions.

The firms' reliance on growth options whereby the negative jump risk channel takes effect on firm valuations and firm betas is a crucial feature of the model. For our last set of empirical analysis, we compare the strength of the high distress-small growth relation across groups of firms sorted on known empirical proxies for growth intensity (Grullon, Lyandres, and Zhdanov (2010)). The results confirm that the high distress-small growth relation strengthens in the firms' reliance on growth options. Here again, the results are in strong support of the model.

Berk, Green, and Naik (1999) were among the first to establish a linkage between corporate investments and firm betas to explain anomalous regularities in the cross-section of stocks.[3,4] Since then, the literature has been extended in many directions (Carlson, Fisher, and Giammarino (2004); Zhang (2005); Sagi and Seashole (2007); Cooper (2007)). A common theme in this literature focuses on the extent that growth options contribute to the beta of the firm relative to the firm's assets-in-place. In our model, growth options are risker than assets-in-place, hence options have the ability to amplify the firm's beta in line with the extant literature (Carlson, Fisher, and Giammarino (2004)). We add to this literature by expanding the description of the firm's operating environment in an important way to reconcile the distress and the small growth anomalies. While higher distress increases the firm's weight on growth due to a greater growth option value, it nonetheless attenuates the effect that the growth option has on the firm's beta by reducing the option's beta.

Using the empirical proxy for distress of Ohlson (1980), Griffin and Lemmon (2002) show that the low average stock returns of high distress firms first documented by Dichev (1998) is concentrated among firms with growth characteristics; attributes not traditionally associated with high distress (Fama and French (1996); Vassalou and Xing (2004)). Consequently, the literature has attributed the distress anomaly to market mispricings (Griffin and Lemmon (2002)), and to investors' strong preference for glamor stocks (Conrad, Kapadia, and Xing (2012)). The growth characteristics of high distress firms motivates our own option-based explanation for the anomalies. Our explanation, however, departs from market mispricing and suggests that the empirical regularities are not anomalous relative to the correct asset pricing model.

The empirical literature highlights that corporate failures are idiosyncratic events (Opler and Titman (1994); Asquith, Gertner, and Sharfstein (1994)) motivating our assumption for distress as a non-systematic risk. Jump risk imparts differentials in expected stock returns

[3] Fama and French (1992) provide evidence on the ability of size and book-to-market to explain returns. Fama and French (1996) provide a cross-sectional landscape view of how average returns vary across stocks. Anderson and Garcia-Feijoo (2006) offer empirical evidence on the relation between corporate investments and average returns.

[4] Firm-level investment in a real option context was first pioneered by MacDonald and Siegel (1985), MacDonald and Siegel (1986) and Brennan and Schwartz (1985), and later adopted and extended by many others. Dixit and Pindyck (1994) is a standard reference for a detailed analysis of the literature.

by relating inversely with firm maturity. This feature of the model shares similarities with Berk, Green, and Naik (1999) and Bena and Garlappi (2012), among others, in that a firm-specific characteristic can contribute to differentials in expected returns through investment policies that depend on a firm-specific attribute. Furthermore, based on our own empirical results, and others' (Conrad, Kapadia, and Xing (2012)), on the weak relation between financial leverage and high distress, we model distress as an event that does not necessarily relate to financial distress.[5] Letting downward jump risk to proxy for distress allows us to develop a more parsimonious model and to focus on a novel mechanism for firm valuation and firm beta. Our model is well-grounded to the extent that distress risk proxies for the possibility of sizeable losses in firm value.

We are also motivated by a growing literature devoted to the intersection of high distress and glory stocks. Grounded on the findings that many glory-predicted stocks are the stocks of highly distressed firms with virtually zero financial leverage, Conrad, Kapadia, and Xing (2012) conclude that the distress anomaly is an artifact of the market's overvaluation of glory-predicted stocks. Since glory-predicted stocks share similar firm-characteristics as small growth and high distress stocks (Conrad, Kapadia, and Xing (2012)), we contribute to this line of research with a potentially new explanation for the anomalously low average returns of glory-predicted stocks.

To our knowledge little inroads have been made to explain the distress anomaly or the small growth anomaly in a rational framework (Garlappi and Yan (2011); George and Hwang (2010)); a void we hope to fill with this paper. Garlappi and Yan (2011) offer an explanation for the distress anomaly based on the shareholder's ability to extract firm value through strategic default on the firm's debt resulting in a lower equity beta in face of financial distress. George and Hwang (2010) offer an alternative explanation that hinges on the notion that low beta firms optimally utilize greater financial leverage than high beta firms. We contribute to this line of research with a novel explanation. Our explanation

[5] Whenever appropriate, we report our empirical results in relation to the firms' financial leverage ratio.

focuses on the ability of non-systematic jump risks to attenuate the leverage-enhancing effects of growth options on the risk of the firm, and distress in our model takes on a broader meaning than financial distress. Consequently, the economic forces driving the results of our model are different from Garlappi, Shu, and Yan and George and Hwang.[6] Additionally, our model produces a direct correspondence between high distress and small growth in the cross-section of firms, contributing to this literature with a novel explanation for the small growth anomaly.

The rest of the paper is organized as follows. The next section builds the model and develops the main ideas. Section 3 explains the empirical study. The last section concludes. An appendix with technical details accompanies the paper.

2 Model

In this section, we develop the model and discuss its properties.

2.1 The Environment

We augment the growth option model of Carlson, Fisher, and Giammarino (2004) by incorporating risk of downward jump in asset value. Firms gain incremental access to the product market through irreversible investments in physical capital in each stage of the firms' life-cycle until they reach full maturity. In each stage i, firms can investment an amount I_i in order to advance to the next stage $i+1$ and increase production scale from ξ_i to ξ_{i+1}, where $\xi_{i+1} > \xi_i$. Firm stage proxies for the firms' maturity and we assume that there are in total n stages in the firms' life-cycle. Since investment in capital is irreversible, ξ_i also proxies for the level of capital stock.

[6] Our departure from Garlappi, Shu, and Yan (2008) and George and Hwang (2010) offers more than an alternative explanation for the distress anomaly. One challenge shared by the aforementioned explanations is their reliance on the premise that high distress is synonymous with financial distress; a feature that is inconsistent with the data. We argue that operating distress is the likely culprit for the high distress status of the largest segment of the firms with high O-Scores.

Each firm produces a single commodity that can be sold in the product market at price P_i. The price is composed of three components

$$P_i = XYZ_i \tag{2.1}$$

where X, Y and Z_i are respectively the idiosyncratic, systematic and the jump shock variables. The time and firms' subscripts are omitted for convenience. The profitability shocks have the following dynamics:

$$\frac{dX}{X} = \sigma_X dB_1 \tag{2.2}$$

$$\frac{dY}{Y} = \mu dt + \sigma_Y dB_2 \tag{2.3}$$

where dB_1 and dB_2 are the increments of two independent Brownian motions. We assume that the market-determined required rate of return μ on the commodity is justified by its systematic risk σ_Y.

Following the approach used in numerous intensity-based models in the credit risk literature (Duffie and Singleton (2003)), we model distress risk in reduced-form by allowing the possibility of a sudden loss in asset value.[7] To this end, we assume that in each period, a stage i firm may encounter a downward jump in the value of its assets proxied by $z_i = 1$ which occurs with probability λ_i per unit of time. Distress may arise as the outcome of financial insolvency, from poor operating performance unrelated to debt obligations such as defeat in a patent race, closure from the inability to make profits due to excessive regulation or competition, bad management, sudden technological or output obsolescence, or the inability to pay suppliers, taxes, wages or pension, among others. Motivated by our own empirical findings discussed in the sequel, we assume that λ_i is decreasing in maturity, i.e., $\lambda_i > \lambda_{i+1}$.

[7] Our model is well-grounded to the extent that distress risk proxies for the possibility of sizeable losses in firm value.

We distinguish the main driving force of our model from the shareholder recovery feature of Garlappi and Yan (2011) in order to focus on a novel mechanism for the relation between distress and returns. We assume that if a jump occurs, then the firm ceases to operate and the firm value is reduced to zero, leaving shareholders with zero recovery.[8] More specifically, conditional on $z_i = 0$

$$\frac{dZ_i}{Z_i} = \lambda_i dt - dz_i \tag{2.4}$$

where

$$dz_i = \begin{cases} 0 & \text{, with prob. } (1 - \lambda_i)dt \\ 1 & \text{, with prob. } \lambda_i dt \end{cases} \tag{2.5}$$

Motivated empirically by Opler and Titman (1994) and Asquith, Gertner, and Sharfstein (1994) on business failures, we assume that the jump risk is non-systematic, therefore it does not command a risk premium, i.e., $E\left[\frac{dZ_i}{Z_i}\right] = 0$.

Using Ito's Lemma, we can write the price process as

$$\frac{dP_i}{P_i} = (\mu + \lambda_i)dt + \sigma_X dB_1 + \sigma_Y dB_2 - dz_i \tag{2.6}$$

Equation (2.6) shows that P_i follows a jump-diffusion process. The drift of the price incorporates the jump risk λ_i. Everything else the same, incorporating a non-systematic risk of abruptly taking a negative jump is accompanied by a commensurate increase in the drift of the commodity price process in order for the market-determined expected rate of return on the commodity to remain constant, i.e., $E\left[\frac{dP_i}{P_i}\right] = E\left[(\mu + \lambda_i)dt\right] + E\left[\sigma_X dB_1\right] + E\left[\sigma_Y dB_2\right] - E\left[dz_i\right] = (\mu + \lambda_i)dt - \lambda_i dt = \mu dt$.[9]

Following Carlson, Fisher, and Giammarino (2004), in each period firms have fixed operating costs f_i which is strictly increasing in the stage of the firms' life cycle, i.e. $f_{i+1} > f_i$ for $1 \leq i < n$. The profit per unit of time for a firm in stage i is

[8] Zero shareholder recovery upon the arrival of a jump is not necessary in our model. Our model only requires that jumps be downward and non-systematic.

[9] The price process (2.6) follows a similar process as Merton (1976). Merton develops an option pricing model with non-systematic jump risk.

$$\pi_i(P_i) = \xi_i P_i - f_i \tag{2.7}$$

The uncertainty in the price of the commodity drives all of the operating uncertainty of the firm. Investors in the stock market can hedge systematic uncertainty in the firms' operations by investing in two assets. Let M_t denote the price of a riskless bond with dynamics

$$\frac{dM}{M} = rdt \tag{2.8}$$

and S denote a risky asset with dynamics

$$\frac{dS}{S} = \mu_S dt + \sigma_S dB_2 \tag{2.9}$$

Asset S has a beta of one, therefore $\lambda = \frac{\mu_S - r}{\sigma_S}$ is the market price of risk. The proportion of S held in a replicating portfolio determines the beta of the portfolio. This greatly simplifies firm valuation and the determination of the firms' beta.

Given the structure of the environment, a conditional version of the CAPM holds in the model. The instantaneous expected return for the equity of a stage i firm is given by

$$E\left[\frac{\pi_i(P_i)dt + dV_i(P_i)}{V_i(P_i)dt}\right] = r + \beta_i(P_i)\sigma_S\lambda \tag{2.10}$$

where $\beta_i(P_i)$ represents the firm's beta.

2.2 Firm Valuation

We describe the valuation of the firms in our model. The market value of a stage i firm is given by

$$V_i(P_i) = E^{\mathbb{Q}}\left[\int e^{-r}\pi_i(P_i)dt\right] \tag{2.11}$$

where $E^{\mathbb{Q}}[.]$ denotes the expectation under the $\mathbb{Q}$ measure. The appendix shows that the market value of the firm can be found as the solution of the following differential equation

$$\frac{1}{2}P_i^2\sigma^2 V_i''(P_i) + (\mu + \lambda_i - \sigma_Y \lambda)\, P_i V_i'(P_i) + \pi_i(P_i) = (r + \lambda_i)V_i(P_i) \tag{2.12}$$

given the appropriate boundary and optimality conditions, which is summarized in the following proposition:

Proposition 1 *If the price process is given by (2.6), then the market value of a stage i firm is*

$$V_i(P_i) = \begin{cases} AP_i(P_i) & , \text{ if } i = n \\ AP_i(P_i) + GO_i(P_i) & , \text{ if } 1 \leq i < n \end{cases} \tag{2.13}$$

where $AP_i(P_i)$ denotes the market value of the firm's assets-in-place

$$AP_i(P_i) = \frac{\xi_i P_i}{r + \lambda_i - \mu^*} - \frac{f_i}{r + \lambda_i} \tag{2.14}$$

for $\mu^* = \mu - \sigma_Y \lambda < r + \lambda_i$, and $GO_i(P_i)$ denotes the market value of the firm's growth option

$$GO_i(P_i) = B_i P_i^{\phi_i} = \left(\frac{P_i}{\underline{P_i}}\right)^{\phi_i} \left[V_{i+1}(\underline{P_i}) - AP_i(\underline{P_i}) - I_i\right] \tag{2.15}$$

where $\phi_i = \frac{1}{2} - \frac{\mu^* + \lambda_i}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu^* + \lambda_i}{\sigma^2}\right)^2 + \frac{2(r+\lambda_i)}{\sigma^2}}$, and $\underline{P_i}$ is the optimal threshold for P_i when the firm in stage i chooses to advance to stage $i + 1$

For a stage $1 \leq i < n - 1$ firm, $\underline{P_i}$ is the solution to the following equation

$$A_i + B_i \beta_i \underline{P_i}^{\phi_i - 1} = A_{i+1} + B_{i+1}\phi_{i+1}\underline{P_i}^{\phi_{i+1}-1} \tag{2.16}$$

where $A_i = \frac{\xi_i}{r+\lambda_i - \mu^*}$, $F_i = \frac{f_i}{r+\lambda_i}$, and B_i takes the recursive expression

$$B_i = (A_{i+1} - A_i)\, \underline{P_i}^{1-\phi_i} + \underline{P_i}^{-\phi_i} \left(B_{i+1}\underline{P_i}^{\phi_{i+1}} - F_{i+1} + F_i - I_i\right) \tag{2.17}$$

For a stage $i = n-1$ firm, $\underline{P_i}$ has a closed-form solution

$$\underline{P_i} = \frac{\phi_i}{\phi_i - 1} \times \frac{F_{i+1} - F_i + I_i}{A_{i+1} - A_i} \tag{2.18}$$

Proof: *See Appendix.* ∎

Similarly to Carlson, Fisher, and Giammarino (2004), the valuation expression for a stage $i < n$ firm contains two components. The first is the value of the firm's existing operations, which is the value of a growing perpetuity generated by the revenues minus the value of future fixed operating charges. The second is the value of the growth option. The value of a fully mature firm is entirely composed of the firm's existing operations since mature firms do not have an option to expand.

Not present in Carlson, Fisher, and Giammarino (2004), but present in this model, is the dependence of the firm value on jump risk λ_i. Including the jump risk results in a commensurate increase in the discount rate used in the valuation of the firm. Since call option values are increasing in the discount rate (Hull (2011)), incorporating downward jumps in the model produces a greater growth option value, and the optimal investment threshold $\underline{P_i}$ reflects this dependency on λ_i.

2.3 Distress and Firm Beta

We now focus on the determination of firm risk. The appendix shows that the beta of a firm is given by $\beta_i(P_i) = \frac{V_i'(P_i)P_i}{V_i(P_i)} \frac{\sigma_Y}{\sigma_S}$. Using the expressions in Proposition 1, we obtain the following proposition:

Proposition 2 *If the market value of the firm is given by (2.13), then the firm's beta is given by*

$$\beta_i(P_i) = \begin{cases} \left[1 + \frac{F_i}{V_i(P_i)}\right] \frac{\sigma_Y}{\sigma_S} & , \text{ if } i = n \\ \\ \left[1 + (\phi_i - 1)\frac{GO_i(P_i)}{V_i(P_i)} + \frac{F_i}{V_i(P_i)}\right] \frac{\sigma_Y}{\sigma_S} & , \text{ if } 1 \le i < n \end{cases} \tag{2.19}$$

where F_i, $GO_i(P_i)$ and ϕ_i are expressions given in Proposition 1.

Proof: *See Appendix.* ∎

The beta of the firm is a weighted average of the betas of the firm's assets. Mature firms do not have any growth options, therefore the firm's beta is made up entirely of the firm's operations. $\frac{\sigma_Y}{\sigma_A}$ captures the systematic risk in the product market, hence, the beta of the firm's revenue is given by $\frac{\sigma_Y}{\sigma_A}$. Risk from operating leverage is captured by $\frac{F_n}{V_n(P_n)}\frac{\sigma_Y}{\sigma_S}$. Since profits include both revenues and fixed operating charges, the beta of a mature firm is given by $\left[1+\frac{F_n}{V_n(P_n)}\right]\frac{\sigma_Y}{\sigma_S}$. By contrast, stage $i < n$ firms have growth options. Growth options are riskier than assets-in-place, i.e., $\phi_i > 1$, thus the beta of the firm incorporates the incremental risk of the growth option given by $(\phi_i - 1)\frac{\sigma_Y}{\sigma_S}$, weighted by the percentage of total firm value in the growth option $\frac{GO_i(P_i)}{V_i(P_i)}$.

Building on Carlson, Fisher, and Giammarino (2004), the model is also able to generate the separate size and book-to-market effects in the cross-section of stock returns. Irreversibility in physical capital adds to operating leverage, while limits to growth adds to the importance of growth options for firm risk. When demand for a firm's product decreases, equity value falls relative to book value, proxied by the capital stock, and the riskiness of returns increases due to a greater operating leverage generating the book-to-market effect. Firm size, on the other hand, captures the importance of growth options relative to assets-in-place contributing to the size effect. While revenue betas are assumed constant, firm betas are nonetheless time-varying and reflect past expansions as well as current demand.

Not present in Carlson, Fisher, and Giammarino (2004), but present in this model, is the dependence of the firm's beta on λ_i. We summarize this feature of the model in the next proposition.

Proposition 3 *Growth as a proportion of total firm value is increasing in λ_i, i.e.,*

$$\frac{d}{d\lambda_i}\left[\frac{GO_i(P_i)}{V_i(P_i)}\right] > 0, \tag{2.20}$$

and the impact of an increase in λ_i on the firm's beta attributed to the size effect $\beta_i^{size} = (\phi_i - 1)\frac{GO_i(P_i)}{V_i(P_i)}$ *is given by the following relation*

$$\frac{d}{d\lambda_i}\left[\beta_i^{size}(P_i)\right] = \frac{d}{d\lambda_i}\left[(\phi_i - 1)\frac{GO_i(P_i)}{V_i(P_i)}\right] = \underbrace{\frac{GO_i(P_i)}{V_i(P_i)}\frac{\partial\phi_i}{\partial\lambda_i}}_{<0} + \underbrace{\frac{d}{d\lambda_i}\left[\frac{GO_i(P_i)}{V_i(P_i)}\right](\phi_i - 1)}_{>0} \quad (2.21)$$

Proof: *See Appendix.* ∎

Proposition 3 highlights the central ideas of our paper. Given a fixed F_i, a greater λ_i results in an increase in the value of the growth option and a greater percentage of total firm value in the growth option, reducing the book-to-market ratio and operating leverage. This result is closely related to a little-known and counterintuitive property of option pricing first formally shown by Merton (Merton (1976)). Everything else the same, a call option is more valuable if the underlying stock has a non-systematic risk of taking a downward jump. In a similar vein, incorporating downward jump risk in an otherwise standard model of corporate investments produces greater growth option values and reduced the operating leverage ratio $\frac{F_i}{V_i(P_i)}$.[10]

Insert Table 1 Here

Table 2 reports numerical values of B_i, $\underline{P_i}$ and $\frac{F_i}{B_i}$ for a stage $i = 1$ firm and various values of λ_i based on model parameter values summarized in Table 1. B_i is the constant of integration that determines the value of the growth option, hence $\frac{F_i}{B_i}$ proxies for operating leverage and the book-to-market ratio. Consistent with the proposition, a greater λ_i increases B_i, which coincides with a higher $\underline{P_i}$.[11] Given a fixed F_i, a greater λ_i reduces $\frac{F_i}{B_i}$.

[10] Integrating a non-systematic risk of abruptly realizing a negative jump in the value of the firm's assets implies a commensurate increase in the drift of the firm's underlying shock variable in order for its market-determined expected rate of return to remain constant. This comprises a condition for a greater growth option value.

[11] A larger option value implies a greater opportunity cost of exercising the option, hence a higher $\underline{P_i}$ (Dixit and Pindyck (1994)).

In sum, the proposition establishes a linkage between high distress, a high ratio of market to book value, and low expected returns.

Insert Table 2 Here

The proposition also highlights the impact that λ_i has on the size effect of Carlson, Fisher, and Giammarino. A greater λ_i affects the size effect in two opposing ways. On the one hand, λ_i weakens the impact that the growth option has on firm risk, i.e., $\frac{GO_i(P_i)}{V_i(P_i)}\frac{\partial\phi_i}{\partial\lambda_i} < 0$, since a greater option value implies a lower systematic component of total firm value and a lower beta, i.e, $\frac{\partial\phi_i}{\partial\lambda_i} < 0$. On the other hand, λ_i increases risk due to a greater percentage of total firm value in growth, i.e., $\frac{d}{d\lambda_i}\left[\frac{GO_i(P_i)}{V_i(P_i)}\right](\phi_i - 1) > 0$, since growth options are risker than the assets-in-place, i.e, $\phi_i > 1$. Numerical values in table 2 show that the net effect is to reduce both β_i^{size} and β_i.

Taken together, Proposition 3 highlights that growth options and high distress, proxied by a high jump risk, generate low expected equity returns and a large ratio of market to book value. We verify this linkage empirically in the next section.

3 Empirical Analysis

In this section, we test the predictions of the model and show empirical support in the data.

3.1 Data Source

All our accounting variables are from the annual COMPUSTAT data files. Following Dichev (1998), our accounting variable sample starts from year 1980.[12] All our market-related variables are from CRSP monthly return files, with the exception of monthly factor returns and

[12]COMPUSTAT data for the construction of our distress variable is most reliable for observations starting in 1980.

risk-free rates, which are from Ken French's website.[13] Returns are adjusted for delisting, and we drop from our sample stocks of firms with a negative book-to-market ratio. It is common in the empirical literature to exclude stocks with prices below $1 to remove the effects of illiquidity. Low-priced stocks on average have greater risk of distress (Garlappi and Yan (2011)), consequently are likely to experience greater failure risk. Therefore the reported results are based on the full sample without a minimum price filter, but our results are robust to the exclusion of stocks with price below $1. We consider only ordinary shares traded on the NYSE, AMEX and Nasdaq with primary link to companies on COMPUSTAT with US domestic data source. Our baseline sample contains 1,026,726 firm-month stock return observations with non-missing distress variable and spans from July 1981 to December, 2010.

3.2 Variable Description

We require several firm characteristics in order to investigate the relation between default and small growth. Following many in the literature, we use the firms' market equity capitalization to proxy for firm market size, and the firms' book-to-market ratio to proxy for value or growth.[14] An assumption of our model is λ_i that relates inversely with firm maturity. We use firm age, defined as the number of years since the firms' first stock return observation on CRSP, as a proxy for firm maturity to verify this assumption.

We also require an empirical proxy for λ_i. To this end we follow Dichev (1998), Griffin and Lemmon (2002) and George and Hwang (2010), among many others, and use the likelihood of corporate failure of Ohlson (1980) (O-Score). We follow (Dichev (1998)) and

[13] http:// mba.tuck.dartmouth.edu/ pages/faculty/ken.french/data library.html

[14] Following Fama and French (1993), market value of equity is defined as the share price at the end of June times the number of shares outstanding. Book equity is stockholders' equity minus preferred stock plus balance sheet deferred taxes and investment tax credit if available, minus post-retirement benefit asset if available. If missing, stockholders' equity is defined as common equity plus preferred stock par value. If these variables are missing, we use book assets less liabilities. Preferred stock, in order of availability, is preferred stock liquidating value, or preferred stock redemption value, or preferred stock par value. The denominator of the book-to-market ratio is the December closing stock price times the number of shares outstanding.

compute O-Scores according to the following formula

$$
\begin{aligned}
O-Score_t = & -1.32 - 0.407 \times log(TotalAssets_t) + 6.03 \times \frac{TotalLiabilities_t}{TotalAssets_t} \\
& - 1.43 \times \frac{WorkingCapital_t}{TotalAssets_t} + 0.076 \times \frac{CurrentLiabilities_t}{CurrentAssets_t} \\
& - 1.72 \times (1 \text{ if } TotalLiabilities_t > TotalAssets_t, \text{ 0 otherwise}) \\
& - 2.37 \times \frac{NetIncome_t}{TotalAssets_t} - 1.83 \times \frac{FundsFromOperations_t}{TotalLiabilities_t} \\
& + 0.285 \times (\text{1 if Net Loss for the last 2 yrs, 0 otherwise}) \\
& - 0.521 \times \frac{NetIncome_t - NetIncome_{t-1}}{|NetIncome_t| + |NetIncome_{t-1}|}
\end{aligned}
$$

As shown by Ohlson, the first four inputs of the O-Score capture the firms' financial state, while the last five inputs capture operating performance. This feature of the O-Score is suitable for our study because distress in our model incorporates both failure from financial distress and poor operating performance. To distinguish the effects of poor operating performance from financial distress, we present our results for firms with different levels of financial leverage whenever appropriate.

We compute a credit measure based on the issuers' S&P credit rating for descriptive purposes. Following Avramov, Chordia, Jostova, and Philipov (2012), we transform COMPUSTAT S&P issuer ratings into numerical values as follows: $AAA = 1, AA+ = 2, AA = 3, AA- = 4, A+ = 5, A = 6, A- = 7, BBB+ = 8, BBB = 9, BBB- = 10, BB+ = 11, BB = 12, BB- = 13, B+ = 14, B = 15, B- = 16, CCC+ = 17, CCC = 18, CCC- = 19, CC = 20, C = 21, D = 22$. A higher credit score corresponds to a lower credit rating.

We require empirical proxies for growth option intensity when analyzing the high distress-small growth relation in the cross-section. We follow Grullon, Lyandres, and Zhdanov (2010) in the selection of our main growth option variables. The most common type of real options come in the form of future growth opportunities (Grullon, Lyandres, and Zhdanov (2010); Brennan and Schwartz (1985); MacDonald and Siegel (1986); Majd and

Pindyck (1987); Pindyck (1988)). We consider firm size as an inverse measure of growth opportunities because larger firms tend to be more mature and have larger proportions of their values from assets-in-place, while smaller firms tend to derive value from future growth opportunities (Brown and Kapadia (2007); Carlson, Fisher, and Giammarino (2004)). We define firm size as the value of total assets as recorded on COMPUSTAT.

Our second inverse proxy for growth options is firm age. Older and more established firms tend to derive larger proportions of profits and firm value from assets-in-place (Lemmon and Zender (2010); Carlson, Fisher, and Giammarino (2004)), while younger and more infant firms tend to derive a larger portion of firm value from growth options. Age is defined as the difference between the month of the return observation and the month in which the stock first appeared on CRSP monthly return files.

Growth opportunities are revealed in growth capitalized in the future in the form of increased sales. Therefore, for our third growth opportunity variable, we define sales growth as the sum of the sales growth rates starting 2 years and ending 5 years after the stock return observation.[15] Following Grullon, Lyandres, and Zhdanov (2010), we alleviate concerns of spurious correlations between contemporaneous surprises in growth and monthly returns by merging month t returns with sales growth starting two years following the return observation.

The fourth and last growth option measure is R&D intensity. Research and development generates investment opportunities. Therefore, the greater the firm's R&D expenditure the more growth options the firm is expected to have. R&D intensity is defined as the ratio of annual R&D expenditures and the beginning-of-year R&D capital where we follow Chan, Lakonishok, and Sougiannis (2001) in the definition of R&D capital.

We match returns from January to June of year t with year $t-2$ accounting variables from COMPUSTAT, while the returns from July until December are matched with COM-

[15] One caveat with this growth variable is the possibility of look-ahead bias. As in Grullon, Lyandres, and Zhdanov (2010), we are not concerned with potential issues related to look-ahead bias since the focus of our paper is on investigating the relation between distress and small growth returns, and not on predicting future stock returns.

PUSTAT variables of year $t-1$. This matching scheme is used for all our matches involving market-related CRSP variables and COMPUSTAT variables, except for sales growth whose matching was explained previously. Matching this way is conservative and ensures that the observable firm characteristics are contained in the information set prior to the realization of stock returns.

3.3 Descriptive Analysis

In this section, we report some descriptive statistics for groups of firms sorted on O-Scores, and groups of firms sorted on size and book-to-market ratio. The purpose of this section is to highlight commonalities in some key firm-characteristics between high distress and small growth firms in line with our model predictions.

At the end of each June, we group stocks evenly into deciles based on O-Score values, and independently, we sort and rank stocks into 25 5×5 groups based on size and book-to-market ratio. Following most in the literature, the quintile cutoff values for size and book-to-market are based on NYSE stocks. Then for each group of O-Score, and for each 5×5 size and book-to-market groups, we compute the sample mean values of the following: O-Scores, credit score, age, market equity, book-to-market, book leverage, market leverage, number of stock-month observations (N) with non-missing O-Scores, and the percentage of the firms with book and market financial leverage in the bottom and the top 30th percentile of the sample. Tables 4 and 5 summarize the results.

Insert Table 4 Here

Panel A of Table 4 shows that the highest O-Score decile correlates with the smallest mean market equity value. It also has the second lowest mean book-to-market ratio which is significatively below the full sample mean, highlighting that the most distressed firms command market valuations relative to book values comparable with operationally and financially sound firms. This suggests the presence of a linkage between high distress and small firms with low book-to-market ratios in the cross-section. This feature of the data breaks away from the commonly held view in the literature that a high book-to-market ratio captures high distress risk.[16] The table also shows that the top O-Score decile group has the lowest average firm age and the worst average credit score. This points to the existence of an inverse relation between high distress and firm maturity, motivating the inverse relation between λ_i and firm maturity in our model.

Insert Table 5 Here

Table 5 reports summary statistics for each of the 5×5 size and book-to-market ratio firms. The intersection of the smallest and the lowest book-to-market ratio shares similar firm-characteristics as the top O-Score decile. It has the highest mean O-Score, the worst mean credit score, and the lowest mean age among all the 5×5 size and book-to-market ratio firms. This offers further verification that high distress correlates inversely with firm size and firm maturity. These results also confirm that high distress correlates positively with market valuations relative to book values; results that are consistent with our model, but defy the conventional wisdom that value relates to high distress.

Next, we investigate if poor operating performance, rather than financial distress, is the likely contributor to high distress. The extant explanations in the literature have used features that rely on the presence of high financial leverage or high financial distress to explain the distress anomaly. These features are not consistent with the data. Table 4 shows that the highest O-Score decile group has a mean book financial leverage slightly above the full sample average and a mean market financial leverage significantly below the full sample average. The top O-Score decile has a mean financial leverage comparable with the middle O-Score decile, highlighting that firms with the most distress have financial

[16]The literature (Fama and French (1996); Vassalou and Xing (2004)) views that the value premium may be compensation for distress risk.

leverage comparable with operationally and financially sound firms. Since firms with low financial leverage are unlikely to suffer from high financial distress, this evidence suggests that it is poor operating performance, not financial distress, the likely source for the distress anomaly.

The same conclusion applies to the intersection of the smallest and the lowest book-to-market ratio firms. This group of firms has a mean book financial leverage slightly below the full sample average and a mean market financial leverage significantly below the full sample average, pointing to the conclusion that small growth firms also have mean financial leverage comparable with operationally and financially sound firms. Therefore, poor operating performance, rather than financial distress from high financial leverage, seems to be the contributor to the high mean O-Scores for the small growth firms.

To further investigate the firms' financial leverage, each June we sort the firms based on book leverage and, separately, on market leverage and compute the top and bottom 30th percentile cutoff points. Then for each O-Score and for each size and book-to-market ratio classifications, we investigate the proportion of firms that had financial leverage in the top 30th and the bottom 30th percentile of the full sample. Panels B and C of Table 4 report these mean proportions using book leverage and market leverage respectively for O-Score deciles firms and Panels I to L of Table 5 report mean proportions for the size and book-to-market ratio firms.

On average 32% and 15% of the highest O-Score decile firms had financial leverage in the top 30th percentile based on book and market leverage respectively, but a mean 32% and 56% of these firms had financial leverage in the bottom 30th percentile. While some high O-Score firms had heavy exposure to borrowing, a much larger subset of the high O-Score firms had very low financial leverage, further highlighting that poor operating performance is the main contributor to their high O-Scores.

Lastly, Table 5 shows that small growth firms share similar financial leverage characteristics as the top O-Score firms. On average, only 19% and 3% of the lowest size and book-to-market firms had book and market leverage, respectively, in the top 30th percentile of the full sample, while 37% and 80% had leverages in the bottom 30th percentile. These findings also favor poor operating performance as the main reason for the high mean O-Scores for this group of firms.

Taken together, this section reports findings that break away from the commonly held view that a high book-to-market ratio, or value, proxies for high distress risk, and supports the need for an alternative explanation in which low book-to-market ratios and other growth characteristics – such as young firm age, low financial leverage and high valuations ratios – relate to high distress, in line with the main features of our model. We explore this possibility more closely in the sequel with asset pricing tests. We also reiterate that it is poor operating performance, rather than distress from high financial leverage, the likely contributor to the high distress of the top O-Score and small growth firms.

3.4 Small Growth and Distress Portfolio Returns

In this section, we conduct standard asset pricing tests based on portfolio returns and show that low average stock returns is concentrated among the most distressed firms (distress anomaly), and small firms with low book-to-market ratio (small growth anomaly). Similar results have been shown to exist separately in the literature. The next section further explores the empirical relation between the small growth and the distress anomalies in line with our model's predictions.

The novel contribution of this paper is to highlight that growth options and high distress impart a direct correspondence between high distress and small growth firms in the cross-section. Our model predicts that the low average stock returns of the small firms with low book-to-market ratio should correspond to the low stock returns of high distress firms. If this risk is incorporated in firm valuations by the investors in the market but not completely captured by the existing pricing factors in asset pricing tests, then we should expect pricing errors captured by the intercepts (Jensen's alphas) from portfolio return regressions. This should translate to the portfolio of small growth stocks and the portfolio of the highest O-Score stocks to have large and significantly negative intercepts.

To show this, we adopt the portfolio approach similar to the approach used by Fama and French (1992), and many others. More specifically, at the end of each June, we rank and sort NYSE stocks into 5 groups based on size and, separately, based on book-to-market ratio to determine quintile cutoff values, then 25 monthly value-weighted portfolio returns are computed for each of the 5×5 rank classifications of size and book-to-market for the full sample. Separately, we group stocks evenly into 10 groups based on O-Scores, then we compute monthly value-weighted portfolio returns for each decile. We also construct zero-cost portfolio returns, i.e., portfolios that are long and short the top and bottom quantile portfolios respectively, and returns of portfolios equally weighed the portfolios along each one-way rank classification of size and book-to-market. Then we find the portfolio alphas by estimating the pricing errors relative to the Fama French three factor model (FF-3). More specifically, we fit the following time-series regression

$$r_t - r_{f,t} = \alpha + \gamma_1 SMB_t + \gamma_2 HML_t + \gamma_3 MKTRF_t + \epsilon_t \tag{3.1}$$

where r_t denotes the portfolio return, $r_{f,t}$ is the monthly riskless rate, SMB, HML and $MKTRF$ are the Fama and French (1993) three factors that proxy for size, book-to-market and the market risk premium respectively. When estimating the Jensen's alpha of the zero-cost portfolios, we use the portfolio returns instead of the portfolios' excess returns on the left-hand-side of (3.1).

Insert Table 6 Here

Table 6 reports the estimated pricing errors along with Newey and West (1987) robust t-statistics for the size and book-to-market portfolios. One of the most notable patterns is the positive and significant pricing errors of the zero-cost book-to-market portfolios, or what the literature calls the value premium. More importantly, the table shows that the value premium is not entirely dependent on value alone since it also depends on the low returns of the low book-to-market, or growth, stocks within the sample of the smallest firms. For instance, the positive pricing error of value stocks in the lowest size quintile is less than half as large as the negative pricing error of the growth stocks, i.e., 2.48% vs. -5.56%. The table shows that the value premium is not present in the other four size quintiles. Hence, the value premium is more of an artifact of the abysmally low average returns of the growth stocks rather than the positive average returns of the value stocks among the smallest firms.

The table also shows the size effect for each of the five book-to-market quintiles. The commonly held view that larger stocks earn lower risk-adjusted returns than smaller stocks does not apply to stocks in the first book-to-market quintile. This discrepancy, again, is due to the abysmally negative average returns of the smallest stocks in the lowest book-to-market quintile, underscoring the importance of this group of firms for the cross-section of stock returns.[17]

Insert Table 7 Here

Panel A of table 7 reports the annualized mean returns across O-Score decile portfolios. The table shows a strong inverse relation between O-Score and average stock returns for the three highest O-Score decile portfolios. This pattern is most pronounced and most statistically significant for the top decile portfolio when returns are adjusted for risk using the CAPM, the Fama and French 3-factor or the Carhart 4-factor models, highlighting the robustness of the distress anomaly. Relative to the Fama and French 3-factor model, the stocks with the most distress earn on average a negative risk-adjusted return of 9.28% per annum which comprises a return of -13% in excess of the lowest O-Score portfolio.

[17] Cochrane (2001) attributes the main failure of the Fama and French 3-factor model to price the 25 size and book-to-market portfolios to the excessively low average returns of the small growth stocks.

The return spread between the top and the bottom O-Score decile portfolios is mainly attributed to the low returns of the top O-Score decile portfolio since the lowest O-Score decile portfolio has a mean risk-adjusted return substantially smaller in magnitude (3.74%).

To investigate if financial distress plays a major role in the low average returns, Panels B and C of table 7 report annualized intercept estimates across O-Score decile portfolios for each the financial leverage terciles after stocks are evenly grouped based on O-Score and, separately, on book leverage and market leverage. Panel B shows that the distress anomaly is present in the bottom and middle leverage terciles but not statistically significantly present in the top leverage tercile based on book leverage. Based on market leverage, panel C of the table shows that the distress anomaly is only present in the lowest leverage tercile. The other two leverage terciles lack statistical significance. Hence, the distress anomaly seems much weaker in significance among the higher financial leverage stocks. This is further evidence that favors poor operating performance as the basis for a potential explanation for the distress anomaly.

Overall, the results reported in Tables 6 and 7 demonstrate that small growth stocks, and high distressed stocks have anomalously low average risk-adjusted returns; direction in stock returns in agreement with the predictions of our model. Furthermore, the reported results oppose the premise of high financial leverage as an explanation for the distress anomaly.

3.5 High Distress-Small Growth Relation

The portfolio-based tests in the previous section establish that small stocks with low book-to-market ratio and high distressed stocks share abysmally low average returns. In this section, we investigate if there is a direct correspondence between the small growth anomaly and the distress anomaly.

Proposition 3 of the model suggests that low average stock returns should concentrate among small growth and high distress firms, forming the basis for a relation between the small growth and distress anomalies. To test this prediction, we examine whether the profits earned from a zero-cost investment strategy made up of distress stocks can explain the profits earned from a zero-cost investment strategy made up of small growth stocks. More specifically, the regression specification is

$$SG_t = \alpha + \gamma_1 SMB_t + \gamma_2 HML_t + \gamma_3 MKTRF_t + \gamma_4 DISTRESS_t + \epsilon_t \tag{3.2}$$

where SMB, HML and $MKTRF$ are the Fama and French (1993) three factors defined previously, and SG and $DISTRESS$ are the trading strategy returns based on small growth and high distress respectively. Our model's predictions translate to a positive and statistically significant γ_4 estimate.

At the end of each June, firms are sorted and ranked into 25 5×5 size and book-to-market ratio groups based on NYSE cutoff values, and separately into 10 equally-sized groups based on O-Score, then monthly value-weighted portfolio returns are computed for each group. Then we construct trading strategies using these portfolios. SG is the zero-cost portfolio return that is long the smallest and the lowest book-to-market ratio portfolio and short the middle size and the middle book-to-market ratio portfolio.[18] $DISTRESS$ is the zero-cost portfolio return that is long and short the top and the bottom O-Score decile portfolios, respectively. All the portfolios used in the construction of these strategies are value-weighted and rebalanced monthly. Once the trading strategy returns are computed, we estimate model (3.2).

Insert Table 8 Here

Panel A of Table 8 reports the results. As expected, the estimated loadings of SG on SMB and HML are significantly positive and negative respectively. The loading on $MKTRF$ is very small and insignificant pointing to the conclusion that the SG trading

[18]Our results are robust to different choices for the short portfolio in the SG strategy.

strategy hedges out market risk. More importantly, the estimated loading on $DISTRESS$ is positive and statistically significant, establishing a positive correspondence between the distress and the small growth anomalies in line with our model's predictions. Additionally, the table shows that the small growth anomaly is completely subsumed by the distress strategy. Table 7 shows that the estimated intercept relative to the FF-3 factor model for SG is 13%. By including $DISTRESS$ in the regression reduces the annualized alpha estimate to a statistically insignificant value of 0.7%, pointing to the conclusion that distress is the main contributor to the negative abnormal returns of small growth stocks.

Distress embodies poor operating performance which we argue to be the main reason for the high distress status for most of the high O-Score firms in the cross-section. We expect the relation between the two anomalies to be more cleanly present among low financial leverage firms since a firm's high financial leverage can muddle the effects of high operating distress on the firm's growth options. Panels B and C of Table 8 report the results of regression (3.2) for each financial leverage tercile when leverage is measured by book leverage and market leverage, respectively. Consistent with our conjecture, the table shows that SG has a monotonically decreasing loading on $DISTRESS$ and decreasing statistical significance in terciles of financial leverage, offering further evidence in support of our argument.

Overall, in line with the predications of our model the results reported in Table 8 demonstrate that the anomalies are driven by a common underlying force, and that this source is unlikely to be related to high financial leverage or high financial distress.

3.6 Growth Options and the High Distress-Small Growth Relation

The previous section reports results establishing an empirical correspondence between the distress and the small growth anomalies. The firms' reliance on growth options whereby the negative jump risk channel takes effect on firm valuations and firm betas is a crucial feature of our model. For our last set of empirical analysis, we compare the strength of the high distress-small growth relation across groups of firms sorted on known empirical proxies for growth intensity (Grullon, Lyandres, and Zhdanov (2010)). The growth option intensity variables are firm size (total assets), age, sales growth and R&D expenditure. The construction of the variables is explained in the variable description section of the paper.

At the end of each June, firms are sorted and ranked into 25 5×5 size and book-to-market groups based on NYSE cutoff values, and separately into 10 evenly distributed decile groups based on O-Score, and three evenly distributed terciles based on growth option intensity. Then monthly value-weighted portfolio returns are computed for each group in each growth intensity tercile. We construct SG and $DISTRESS$ trading strategies using these portfolios as before separately for each of the growth option terciles and estimate regression (3.2). The entire procedure is repeated in turn for each growth option intensity measure.

Insert Table 9 Here

Panels A, B, C and D of Table 9 report the regression results when age, size, future sales growth and R&D investment, respectively, proxy for growth intensity. For all the age and size terciles, $DISTRESS$ completely subsumes the anomalously low returns on SG. The table also shows that SG has the largest loading on $DISTRESS$ for the lowest age and the lowest size terciles, while the loading are not large and statistically significant for the other terciles. This establishes that the relation between the small growth and the distress anomalies is stronger for younger and smaller firms.

Panels C and D of the table show consistent results when sales growth or R&D is the proxy for growth intensity. While SG has a positive and statistically significant loading on $DISTRESS$ across all terciles of future sales growth and R&D investment, the estimated loadings are larger for the top tercile groups.

To summarize, the results reported in Table 9 demonstrate that the high distress-small growth relation is driven by a common underlying force that strengthens with the extent to which firm values are reliant on growth options. Here again, the results are in strong support of our model.

4 Conclusion

We propose a new economic mechanism for the anomalously low average equity returns of high distress firms, and small firms with low book-to-market ratio; two anomalies previously shown to exist separately in the literature. We show that growth options and high distress, proxied by the risk of negative jumps in the value of the firms' assets, impart a direct correspondence between high distress firms and small growth firms in the cross-section. The model generates lower betas and greater market values relative to book values for young and growth intensive firms, establishing a direct link between the low average equity returns of high distress firms and small growth firms in the cross-section. We verify this link empirically and offer support for the model.

The previous literature has attributed the low stock returns of high distress and small growth firms to persistent market mispricings, or to investors' preference for glory stocks. By contrast, our explanation appeals for rational expectations and dynamically consistent investment decisions by firms. Our work is part of a growing literature that recognizes the importance of risk-based explanations in addressing seemingly anomalous findings in financial markets, and adds to a better understanding of the main determinants of equity returns in the cross-section firms.

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5 Appendix

This section shows the valuations and the proofs of the propositions stated in Section 2 of the paper.

5.1 Derivation of Valuation Equation

It simplifies valuation if we reexpress the dynamics of the price process (2.6) more concisely by letting

$$\frac{dP_i}{P_i} = (\mu + \lambda_i)dt + \sigma dB - dz_i \tag{5.1}$$

where $dB = \frac{\sigma_X dB_1 + \sigma_Y dB_2}{\sigma}$ and $\sigma = \sqrt{\sigma_X^2 + \sigma_Y^2}$. Then it can be shown that $\mathrm{Cov}\,(dB, dB_2) = \frac{\sigma_Y}{\sigma}dt$, $\mathrm{Cov}\left(\frac{dP}{P}, \frac{dS}{S}\right) = \sigma_S \sigma_Y dt$, and $\rho = \frac{\sigma_Y}{\sigma}$.

Let $U_i(P_i)$ be the value function of an asset that is twice-differentiable in P_i where P_i and z_i follow the processes in equations (5.1) and (2.5) respectively. At this stage, $U_i(P_i)$ can be the value of a growth option or any other non-dividend paying asset. Conditional on $z_i = 0$, the generalized Ito's Lemma (Malliaris (1988)) implies that $U_i(P_i)$ follows the process

$$\frac{dU_i(P_i)}{U_i(P_i)} = \frac{\mu P_i U_i'(P_i) + \frac{1}{2}P_i^2\sigma^2 U_i''(P_i)}{U_i(P_i)}dt + \frac{\sigma P_i U_i'(P_i)}{U_i(P_i)}dB - dz_i \tag{5.2}$$

The first two terms on the right hand side of the equation represent the standard form for Ito's Lemma. The third term represents the jump in the value of $U_i(P_i)$ if default occurs, i.e. $dz_i = 1$. Equation (5.2) can be written more concisely as follows

$$\frac{dU_i(P_i)}{U_i(P_i)} = [\mu_{U_i}(P_i) - \lambda_i \gamma_{U_i}(P_i)]dt + \sigma_{U_i}(P_i)dB + \gamma_{U_i}(P_i)dz_i \tag{5.3}$$

where

$$\mu_{U_i}(P_i) = \left[\frac{(\mu + \lambda_i)P_i U_i'(P_i) + \frac{1}{2}P_i^2\sigma^2 U_i''(P_i)}{U_i(P_i)}\right] + \lambda_i \gamma_{U_i}(P_i) \tag{5.4}$$

$$\sigma_{U_i}(P_i) = \frac{\sigma P_i U_i'(P_i)}{U_i(P_i)} \tag{5.5}$$

$$\gamma_{U_i}(P_i) = -1 \tag{5.6}$$

Following the approach in Merton (1976), we construct a zero market risk hedge portfolio with time varying weights in the traded asset S, asset $U_i(P_i)$ and the riskless asset M. Denote the proportion of a portfolio invested in S, $U_i(P_i)$ and M as w_1, w_2 and $w_3 = 1 - w_1 - w_2$, respectively. The instantaneous rate of return on the portfolio is given by

$$\frac{dW}{W} = w_1 \frac{dS}{S} + w_2 \frac{dU_i}{U_i} + (1 - w_1 - w_2) r dt \tag{5.7}$$

$$= \left[w_1(\mu_S - r) + w_2(\mu_{U_i}(P_i) - r) + r - w_2 \lambda_i \gamma_{U_i}(P_i) \right] dt$$
$$+ w_1 \sigma_S dB_2 + w_2 \sigma_{U_i}(P_i) dB + w_2 \gamma_{U_i}(P_i) dz_i \tag{5.8}$$

where we have substituted (5.3), (2.8) and (2.9) into (5.7) to arrive at (5.8). It is not possible to make this portfolio riskless[19]. Instead, we choose the portfolio weights w_1^* and w_2^* to eliminate market risk only. The default risk in U_i is orthogonal to market risk and perfectly diversifiable and does not command a risk premium (Merton (1976)), consequently, the expected rate of return on the zero market risk portfolio is the risk free rate, r. This implies that

$$w_1^*(\mu_S - r) + w_2^*(\mu_{U_i}(P_i) - r) + r = r \tag{5.9}$$

and

$$w_1^* \sigma_S + w_2^* \sigma_{U_i}(P_i) \frac{\sigma_Y}{\sigma} = 0 \tag{5.10}$$

where we have used the knowledge that $dB = \frac{\sigma_X dB_1 + \sigma_Y dB_2}{\sigma}$ in equation (5.8). Equation

[19] As in Merton (1976), the jump risk in the hedge portfolio is unhedgeable

(5.9) together with equation (5.10) implies that

$$\sigma_S \mu_{U_i}(P_i) = -\frac{r\sigma_Y \sigma_{U_i}(P_i)}{\sigma} + \frac{\sigma_Y \mu_S \sigma_{U_i}(P_i)}{\sigma} + r\sigma_S \tag{5.11}$$

Substituting equations (5.4) and (5.5) into (5.11), and simplifying gives the fundamental valuation equation

$$\frac{1}{2}P_i^2\sigma^2 U_i''(P_i) + (\mu + \lambda_i - \sigma_Y \lambda)\, P_i U_i'(P_i) = (r + \lambda_i)U_i(P_i) \tag{5.12}$$

where we have substituted in the market Sharpe ratio $\lambda = \frac{\mu_S - r}{\sigma_S}$. Given appropriate boundary conditions, equation (5.12) is useful for the valuation of growth options in our paper.

We need to modify (5.12) in order to value dividend paying assets, such as the firms' assets-in-place. To this end, we follow the approach given in Constantinides (1978). The first step in the approach calls for the replacement of the drift of $\frac{dP_i}{P_i}$, $\mu + \lambda_i$, by $\mu^* + \lambda_i = \mu + \lambda_i - \lambda \mathrm{Corr}\left(\frac{dP_i}{P_i}, \frac{dS}{S}\right)\sigma = \mu + \lambda_i - \lambda\rho\sigma = \mu + \lambda_i - \lambda\sigma_Y$. The second step evaluates the stream of cash flows of $U_i(P_i)$ as if the market price of risk were zero, i.e., discount expected cash flows at the riskfree rate.[20] To this end, conditional on $z_i = 0$, the Bellman equation for an asset U_i that pays dividends $\nu_i(P_i)$ over the next instant $\triangle t$ is given by

$$U_i(P_i) = \nu_i(P_i)\triangle t + \frac{1}{1 + r\triangle t}(1 - \lambda_i \triangle t)E\left[U_i(P_i + \triangle P_i)\right] \tag{5.13}$$

One can arrive at the following fundamental valuation equation for $U_i(P_i)$ after multiplying both sides of (5.13) by $1+r\triangle t$, letting $\triangle t$ go to zero, applying Ito's Lemma, and substituting μ by μ^*

$$\frac{1}{2}P_i^2\sigma^2 U_i''(P_i) + (\mu + \lambda_i - \sigma_Y \lambda)\, P_i U_i'(P_i) + \nu_i(P_i) = (r + \lambda_i)U_i(P_i) \tag{5.14}$$

[20] The traded assets M and S allow us to define a new measure under which the process $dB^* = \rho\lambda dt + dB$ is a brownian motion under the $\mathbb{Q}$ measure. For this risk neutral measure, the price dynamics obey $dP_i = (\mu^* + \lambda_i)P_i dt + \sigma P_i dB^*$, where $\mu^* = \mu - \sigma\rho_i\lambda = \mu - \sigma_Y\lambda$. Then the valuation of any asset merely requires that the stream of the asset's cash flows be discounted under the $\mathbb{Q}$ measure. The risk neutral measure is consistent with both valuation approaches considered in this paper.

Equation (5.14) is identical to equation (5.12) except for the $\nu_i(P_i)$ term. Therefore, given appropriate boundary conditions, equation (5.14) is useful for the valuation of either dividend or non-dividend paying assets.

5.2 Proof of Proposition 1: Equity Values

In our model, mature firms do not possess growth options and merely derive value from the perpetual profit flow of their previously deployed assets, or assets-in-place. The value of a mature firm is the solution to equation (5.14) with $U_i(P_i)$ and $\nu_i(P_i)$ replaced by $V_n(P_n)$ and $\pi_n(P_n)$. It can be shown (Dixit and Pindyck (1994)) that the solution is

$$V_n(P_n) = \frac{\xi_n P_n}{r + \lambda_n - \mu^*} + \frac{f_n}{r + \lambda_n} \tag{5.15}$$

where we require that $r + \lambda_n > \mu^* > 0$.

The value of the assets-in-place of a $1 \leq i \leq n-1$ stage firm takes the same functional form as the value of a mature firm. We economize on notation by expressing the value of the assets-in-place of a $1 \leq i \leq n-1$ stage firm as

$$AP_i(P_i) = A_i(P_i) - F_i \tag{5.16}$$

where $A_i(P_i) = \frac{\xi_i P_i}{r+\lambda_i-\mu^*}$ and $F_i = \frac{f_i}{r+\lambda_i}$. The value of a $1 \leq i \leq n-1$ stage firm is given as the sum of the value of the assets-in-place and the value of the growth option $GO_i(P_i)$

$$V_i(P_i) = AP_i(P_i) + GO_i(P_i) \tag{5.17}$$

The value of the growth option satisfies equation (5.14) with $U_i(P_i)$ and $\nu_i(P_i)$ replaced by $V_i(P_i)$ and 0 and the following boundary condition at the exercise threshold $P_i = \underline{P_i}$ (value matching condition)

$$GO_i(\underline{P_i}) = V_{i+1}(\underline{P_i}) - AP_i(\underline{P_i}) - I_i \tag{5.18}$$

It can be shown (Dixit and Pindyck (1994)) that the solution takes the form $GO_i(P_i) = B_i P_i^{\phi_i}$, where $\phi_i = \frac{1}{2} - \frac{\mu^* + \lambda_i}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu^* + \lambda_i}{\sigma^2}\right)^2 + \frac{2(r+\lambda_i)}{\sigma^2}}$. Solving for B_i using the value matching condition (5.18) and substituting it back in gives the value of the growth option

$$GO_i(P_i) = B_i P^{\phi_i} = \left(\frac{P_i}{\underline{P_i}}\right)^{\phi_i} \left(V_{i+1}(\underline{P_i}) - AP_i(\underline{P_i}) - I_i\right) \tag{5.19}$$

The exercise threshold $\underline{P_i}$ must satisfy the optimality condition (smooth pasting condition)

$$GO_i'(\underline{P_i}) = V_{i+1}'(\underline{P_i}) - AP_i'(\underline{P_i}) \tag{5.20}$$

Using the optimality condition, we solve for $\underline{P_{n-1}}$ in closed-form

$$\underline{P_{n-1}} = \frac{\phi_{n-1}}{(\phi_{n-1} - 1)} \frac{(F_n - F_{n-1} + I_{n-1})}{(A_n - A_{n-1})} \tag{5.21}$$

For $1 \leq i < n-1$ stage firms, the optimality condition translates to

$$A_i + B_i \phi_i \underline{P_i}^{\phi_i - 1} = A_{i+1} + B_{i+1} \phi_{i+1} \underline{P_i}^{\phi_{i+1} - 1} \tag{5.22}$$

where $B_i = (A_{i+1} - A_i)\, \underline{P_i}^{1-\phi_i} + \underline{P_i}^{-\phi_i} \left(B_{i+1} \underline{P_i}^{\phi_{i+1}} - F_{i+1} + F_i - I_i\right)$. Since B_i is recursive, the value of a stage i firm and $\underline{P_i}$ must be determined recursively as well. $\blacksquare$

5.3 Proof of Proposition 2: Equity Betas

The (conditional) CAPM beta for any firm can be computed in two different but equivalent ways. In the first approach, we find the firm beta by forming a replicating portfolio with state dependent and time varying weights in the traded assets S and M that exactly reproduces the systematic risk of the firm. The proportion of portfolio value held in S

determines the beta of the firm. To this end, take equation (5.2) and substitute in $dB = \frac{\sigma_X dB_1 + \sigma_Y dB_2}{\sigma}$. By inspection we can see that the diffusion term of the common risk factor can be eliminated by holding $\frac{U_i'(P_i)\sigma_Y P_i}{\sigma_S S}$ units of the stock in the hedge portfolio. Multiplying the number of stocks by S and dividing by $U_i(P_i)$, we get the weight of the hedge portfolio invested in the tradeable asset. Since the tradeable asset has a beta of one, the beta of the firm is given by $\beta_i = \frac{U_i'(P_i)\sigma_Y P_i}{\sigma_S S}\frac{S}{U_i(P_i)} = \frac{U_i'(P_i)\sigma_Y P_i}{\sigma_S U_i(P_i)}$. Substituting in $V_i(P_i)$ from equation (2.13) and its derivative gives (2.19) in the text.

Alternatively, one can find the CAPM beta by computing the firm's return elasticity with respect to the returns of the tradeable asset. The elasticity is $\frac{\mathrm{Cov}[\mathrm{d}V_i(P_i)/V_i(P_i), \mathrm{dS}/S]}{\mathrm{Var}[\mathrm{dS}/S]} = \frac{\sigma_{V,i}\sigma_Y}{\sigma\sigma_S}$. Substituting in (5.5), $V_i(P_i)$ from equation (2.13) and its derivative gives (2.19) in the text. This completes the proof for Proposition 2.[21] ∎

5.4 Proof of Propositions 3: Failure Risk and its Impact on Firm Beta

We pre-compute some derivatives and their signs which will be used to prove proposition 3 in the sequel.

We begin by proving that $\frac{\partial \phi_i}{\partial \lambda_i} < 0$. $\phi_i > 1$ is the positive solution to the following quadratic equation

$$Q(\phi_i) = \frac{1}{2}\sigma^2\phi_i(\phi_i - 1) + (\lambda_i + \mu^*)\phi_i - \lambda_i - r = 0 \tag{5.23}$$

Differentiating the quadratic totally where the derivatives are evaluated at ϕ_i

$$\frac{\partial Q(\phi_i)}{\partial \phi_i}\frac{\partial \phi_i}{\partial \lambda_i} + \frac{\partial Q(\phi_i)}{\partial \lambda_i} = 0 \tag{5.24}$$

[21] There is yet a third approach as shown in Sagi and Seashole (2007). Sagi and Seashole show that the expected excess return is given by $(\mu - \mu^*)\frac{\partial \log V_i(P_i)}{\partial \log P_i} = (\mu - \mu^*)\frac{V_i'(P_i)}{V_i(P_i)}$ where $(\mu - \mu^*)$ is the difference between the unadjusted and risk adjusted mean returns of P_i. In our set up, $(\mu - \mu^*) = \rho\sigma\lambda = \sigma_Y\lambda$. Substituting in $V_i(P_i)$ from equation (2.13), and dividing by $\sigma_S\lambda$ results in the firm beta (2.19) in the text. The beta for a mature firm can be derived similarly.

Since $\frac{\partial Q(\phi_i)}{\partial \phi_i}$ and $\frac{\partial Q(\phi_i)}{\partial \lambda_i}$ are positive, the following must be true

$$\frac{\partial \phi_i}{\partial \lambda_i} < 0 \tag{5.25}$$

Furthermore, the following relations hold

$$\frac{\partial \underline{P_i}}{\partial \phi_i} < 0 \tag{5.26}$$

$$\frac{\partial GO_i(P_i)}{\partial \underline{P_i}} > 0 \tag{5.27}$$

$$\frac{\partial GO_i(P_i)}{\partial \lambda_i} = \left(\frac{P}{\underline{P_i}}\right)^{\phi_i} \left[\frac{\xi_i P_i}{(r+\lambda_i-\mu^*)^2} - \frac{f_i}{(r+\lambda_i)^2}\right] > 0 \tag{5.28}$$

$$\frac{\partial GO_i(P_i)}{\partial \phi_i} = log\left(\frac{P_i}{\underline{P_i}}\right) GO_i(P_i) < 0 \tag{5.29}$$

$$\frac{\partial}{\partial \underline{P_i}}\left[\frac{GO_i(P_i)}{V_i(P_i)}\right] = \frac{\frac{\partial GO_i(P_i)}{\partial \underline{P_i}}\left(V_i(P_i) - GO_i(P_i)\right)}{V_i(P_i)^2} > 0 \tag{5.30}$$

$$\frac{\partial}{\partial \phi_i}\left[\frac{GO_i(P_i)}{V_i(P_i)}\right] = \frac{\frac{\partial GO_i(P_i)}{\partial \phi_i}\left(V_i(P_i) - GO_i(P_i)\right)}{V_i(P_i)^2} < 0 \tag{5.31}$$

$$\frac{\partial}{\partial \underline{P_i}}\left[\frac{F_i}{V_i(P_i)}\right] = \frac{-\frac{\partial GO_i(P_i)}{\partial \underline{P_i}} F_i}{V_(P_i)^2} < 0 \tag{5.32}$$

$$\frac{\partial}{\partial \phi_i}\left[\frac{F_i}{V_i(P_i)}\right] = \frac{-\frac{\partial GO_i(P_i)}{\partial \phi_i} F_i}{V_(P_i)^2} > 0 \tag{5.33}$$

(5.26) is immediate from taking the partial difference of (2.18). (5.27) is trivial since a larger optimal investment boundary implies a larger opportunity cost of investing, hence a larger growth option value. (5.28) follows from $AP_i(P_i) > 0$ and $P_i < \underline{P_i}$. (5.29) is due to $P_i < \underline{P_i}$ and $GO_i(P_i) > 0$. (5.30) and (5.31) are due to (5.27), (5.29) and $V_i(P_i) > GO_i(P_i)$. (5.32) follows from (5.27). (5.33) follows from (5.29).

We now show the proofs. The signs of the expressions in the equations that follow are annotated for convenience.

To prove the first claim of the proposition, we take the derivative of $\frac{GO_i(P_i)}{V_i(P_i)}$ by applying the chain rule, and from (5.35), (5.36) and (5.25) establish the inequality

$$\frac{d}{d\lambda_i}\left[\frac{GO_i(P_i)}{V_i(P_i)}\right] = \underbrace{\frac{\partial}{\partial\lambda_i}\left[\frac{GO_i(P_i)}{V_i(P_i)}\right]}_{>0\ (5.35)} + \underbrace{\frac{d}{d\phi_i}\left[\frac{GO_i(P_i)}{V_i(P_i)}\right]}_{<0\ (5.36)} \underbrace{\frac{\partial\phi_i}{\partial\lambda_i}}_{<0\ (5.25)} > 0 \tag{5.34}$$

where

$$\begin{aligned}\frac{\partial}{\partial\lambda_i}\left[\frac{GO_i(P_i)}{V_i(P_i)}\right] &= \frac{V_i(Pi)\frac{\partial GO_i(P_i)}{\partial\lambda_i} - GO_i(Pi)\frac{\partial V_i(P_i)}{\partial\lambda_i}}{V_i(P_i)^2} \\ &= \frac{\overbrace{\frac{\partial GO_i(P_i)}{\partial\lambda_i}}^{>0\ (5.28)}[V_i(P_i) - GO_i(P_i)] + \overbrace{\frac{\xi_i P_i}{(r+\lambda_i-\mu^*)^2} - \frac{f_i}{(r+\lambda_i)^2}}^{>0 \text{ since } AP_i(P_i)>0}}{V_i(P_i)^2} > 0 \end{aligned} \tag{5.35}$$

and

$$\frac{d}{d\phi_i}\left[\frac{GO_i(P_i)}{V_i(P_i)}\right] = \underbrace{\frac{\partial}{\partial \underline{P_i}}\left[\frac{GO_i(P_i)}{V_i(P_i)}\right]}_{>0\ (5.30)} \underbrace{\frac{\partial \underline{P_i}}{\partial\phi_i}}_{<0\ (5.26)} + \underbrace{\frac{\partial}{\partial\phi_i}\left[\frac{GO_i(P_i)}{V_i(P_i)}\right]}_{<0\ (5.31)} < 0 \tag{5.36}$$

To prove the second claim, again we use the chain rule for the derivative of the size effect on the firm's equity beta with respect to default risk

$$\frac{d}{d\lambda_i}\left[(\phi_i - 1)\frac{GO_i(P_i)}{V_i(P_i)}\right] = \underbrace{\frac{GO_i(P_i)}{V_i(P_i)}}_{>0} \underbrace{\frac{\partial\phi_i}{\partial\lambda_i}}_{<0\ (5.25)} + \underbrace{\frac{d}{d\lambda_i}\left[\frac{GO_i(P_i)}{V_i(P_i)}\right]}_{>0\ (5.34)} \underbrace{(\phi_i - 1)}_{>0} \tag{5.37}$$

∎

Table 1: Model Parameters

This table reports the parameter values used to solve the model developed in Section 2 of the paper.

Model Parameters		
Price Process	Variable Description	Values
μ	Drift term	0.06
σ_X	Idiosyncratic volatility	0.4
σ_Y	Systematic volatility	0.15
λ_1	Probability of failure for stage $i = 1$ firm	$\in [0.005, 0.15]$
λ_2	Probability of failure for stage $i = 2$ firm	0.005
λ_3	Probability of failure for stage $i = 3$ firm	0.001
Firm's Profit Function		
f_1	Fixed production cost for stage $i = 1$ firm	5
f_2	Fixed production cost for stage $i = 2$ firm	20
f_3	Fixed production cost for stage $i = 3$ firm	100
ξ_1	Production scale for stage $i = 1$ firm	1
ξ_2	Production scale for stage $i = 2$ firm	3
ξ_3	Production scale for stage $i = 3$ firm	4
I_1	Investment cost for stage $i = 1$ firm to expand	3
I_2	Investment cost for stage $i = 2$ firm to expand	5
Market Variables		
r	Riskless rate	0.05
μ_S	Drift of tradeable asset (Market)	0.08
σ_S	Diffusion of tradeable asset (Market)	0.2

Table 2: Model Solution: The Dependence of B_i, $\underline{P_i}$, $\frac{F_i}{B_i}$, ϕ_i, β_i^{size} and β_i on λ_i.

The table reports the dependence of B_i, $\underline{P_i}$, $\frac{F_i}{B_i}$, ϕ_i, β_i^{size} and β_i on λ_i for the model developed in Section 2 of the paper. β_i^{size} and β_i are shown for $P_i = 1$ and $i = 1$. A full description of the model parameter values used to solve the model numerically is reported in Table 1.

λ_i	B_i	$\underline{P_i}$	$\frac{F_i}{B_i}$	ϕ_i	β_i^{size}	β_i
0.01	151.29	29.78	0.6	1.09	0.09	1.42
0.02	176.16	30.71	0.4	1.08	0.08	1.22
0.04	187.78	35.37	0.31	1.07	0.07	1.1
0.05	195.61	41.57	0.25	1.07	0.06	1.03
0.07	201.69	49.31	0.21	1.06	0.05	0.99
0.09	206.78	59.09	0.18	1.06	0.05	0.95
0.1	211.2	71.79	0.16	1.05	0.05	0.93
0.12	215.16	89.13	0.14	1.05	0.04	0.91
0.13	218.77	114.64	0.12	1.05	0.04	0.89
0.15	222.13	157.54	0.11	1.04	0.04	0.88

Table 3: Model Solution: The Dependence of Firm Beta on λ_i.

The table reports the dependence of β_i on λ_i for various values of P_i for the model developed in Section 2 of the paper. A full description of the model parameter values used to solve the model numerically is reported in Table 1.

λ_i	P_i					
	0.5	1.4	2.3	3.2	4.1	5
0.01	9.01	1.15	0.97	0.91	0.89	0.87
0.02	2.82	1.05	0.93	0.89	0.87	0.85
0.04	1.89	0.99	0.9	0.87	0.85	0.84
0.05	1.53	0.95	0.88	0.85	0.84	0.83
0.07	1.34	0.92	0.86	0.84	0.83	0.82
0.09	1.22	0.9	0.85	0.83	0.82	0.82
0.1	1.14	0.88	0.84	0.82	0.82	0.81
0.12	1.09	0.87	0.83	0.82	0.81	0.81
0.13	1.04	0.86	0.83	0.81	0.81	0.8
0.15	1.01	0.85	0.82	0.81	0.8	0.8

Table 4: Summary Statistics: O-Score Sorts

Panel A of the table reports the mean O-Score, credit score, age, market equity, book-to-market ratio, book leverage ratio, market leverage ratio, and the mean number of return observations (N) with non-missing O-Scores for each O-Score decile. At the end of each June, stocks are sorted evenly into 10 groups based on O-Score values. We transform COMPUSTAT S&P issuer rating into numerical values to compute the credit scores. A higher credit score corresponds to a lower credit rating. The full mapping is AAA=1, AA+=2, AA=3, AA-=4, A+=5, A=6, A-=7, BBB+=8, BBB=9, BBB-=10, BB+=11, BB=12, BB-=13, B+=14, B=15, B-=16, CCC+=17, CCC=18, CCC-=19, CC=20, C=21, D=22, and follows Avramov, Chordia, Jostova, and Philipov (2012). Firm age is reported in years and it is based on the first month the firm's stock return is available on CRSP. The construction of the other variables is described in the paper. Panels B and C report the mean percentage of the stocks in each O-Score decile with book leverage and market leverage ratios in the bottom and the top 30th percentile of the full sample. The percentile cutoff points are determined at the end of each June after sorting firms based on book leverage and market leverage ratios.

	Full Sample	1	2	3	4	5	6	7	8	9	10
Panel A. Summary Statistics for O-Score Deciles											
O-Score	-0.99	-5.84	-3.55	-2.67	-2.04	-1.51	-1.01	-0.46	0.2	1.2	5.76
Credit Score	9.55	6.68	6.82	7.66	8.71	9.2	9.7	10.43	11.82	13.32	15.15
Age	12.51	11.77	14.52	15.48	15.79	16.44	16.03	14.6	12.68	10.65	7.46
Market Equity	1705.88	3471.85	3544.51	2920.14	2225.63	1711.6	1268.06	830.63	430.63	200.22	84.74
Book-to-market	0.82	0.49	0.67	0.73	0.82	0.89	0.97	1.01	1.06	1.04	0.63
Book leverage	0.44	0.18	0.28	0.35	0.4	0.45	0.49	0.52	0.56	0.59	0.5
Market leverage	0.34	0.08	0.17	0.24	0.31	0.37	0.41	0.45	0.49	0.49	0.32
N	1026726	102831	102660	102689	102661	102607	102732	102693	102657	102692	102504
Panel B. percent of stocks with financial leverage in the bottom 30th percentile for each O-Score Decile											
Book leverage		0.92	0.61	0.34	0.21	0.15	0.12	0.1	0.11	0.13	0.32
Market leverage		0.99	0.9	0.72	0.53	0.37	0.28	0.23	0.22	0.27	0.56
Panel C. percent of stocks with financial leverage in the top 30th percentile for each O-Score Decile											
Book leverage		0	0	0	0.01	0.03	0.05	0.11	0.21	0.33	0.32
Market leverage		0	0	0	0.02	0.03	0.05	0.1	0.18	0.26	0.15

Table 5: Summary Statistics: Size and Book-to-Market Sorts.

This table reports the mean O-Score, credit score, age, market equity, book-to-market ratio, book leverage ratio, market leverage ratio, and the mean number of return observations (N) with non-missing O-Scores, the percentage of firms with book leverage and financial leverage ratios in the bottom and the top 30th percentile of the full sample for each of the 25 5×5 size and book-to-market ratio firms. At the end of each June, NYSE stocks are sorted into 5 groups based on market equity and, separately, based on book-to-market to determine quintile cutoff values. The 30th percentile cutoff points are determined at the end of each June after sorting firms based on book leverage and market leverage ratios. We transform COMPUSTAT S&P issuer rating into numerical values to compute the credit scores. A higher credit score corresponds to a lower credit rating. The full mapping is AAA=1, AA+=2, AA=3, AA-=4, A+=5, A=6, A-=7, BBB+=8, BBB=9, BBB-=10, BB+=11, BB=12, BB-=13, B+=14, B=15, B-=16, CCC+=17, CCC=18, CCC-=19, CC=20, C=21, D=22, and follows Avramov, Chordia, Jostova, and Philipov (2012). Firm age is reported in years and it is based on the first month the firm's stock return is available on CRSP. The construction of the other variables is described in the paper.

	book-to-market					book-to-market					book-to-market				
	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
size	Panel A. O-Score					Panel B. Credit Score					Panel C. Market Equity				
1	1.4	-0.68	-0.87	-0.84	-0.36	14.77	13.82	13.13	12.85	13.57	84.3	88.23	83.93	78.88	58.37
2	-1.97	-2.24	-1.89	-1.53	-1.09	13.61	12.61	11.36	10.67	11.64	390.62	406.5	413.68	426.4	404.99
3	-2.8	-2.43	-1.86	-1.54	-1.26	12.12	10.71	9.82	9.35	10.2	898.36	965.99	952.29	981	899.87
4	-2.99	-2.33	-1.94	-1.6	-1.35	9.52	8.48	8.45	8.63	9.11	2341.05	2318.26	2230.45	2141.41	2182.32
5	-3.08	-2.52	-2.19	-1.94	-1.82	5.82	6.54	6.65	7.11	7.67	23807.3	17330.8	13822.2	10629.5	10312.3
	Panel D. Book-to-Market					Panel E. Book Leverage					Panel F. Market Leverage				
1	0.23	0.48	0.7	0.96	2.04	0.43	0.4	0.41	0.42	0.45	0.18	0.26	0.33	0.4	0.53
2	0.23	0.48	0.69	0.93	1.68	0.38	0.4	0.44	0.47	0.51	0.15	0.26	0.35	0.44	0.57
3	0.23	0.47	0.69	0.95	1.62	0.39	0.43	0.47	0.5	0.53	0.16	0.28	0.38	0.48	0.59
4	0.23	0.47	0.69	0.97	1.54	0.43	0.47	0.51	0.53	0.55	0.18	0.3	0.42	0.51	0.61
5	0.22	0.48	0.71	0.98	1.43	0.49	0.52	0.56	0.58	0.55	0.2	0.36	0.47	0.56	0.6
	Panel G. Age					Panel H. N					Panel I. Book Leverage bottom 30th pctl				
1	6.31	8.37	10.07	11.46	11.8	136692	96670	95982	104129	164715	0.37	0.39	0.35	0.33	0.28
2	7.43	11.58	14.86	16.72	16.41	44737	34813	31314	24992	18167	0.45	0.36	0.27	0.21	0.16
3	10.1	14.98	17.73	19.99	19.84	33860	25023	19619	16295	12127	0.43	0.28	0.2	0.14	0.1
4	14.23	20.22	22.14	23.13	23.24	28786	21079	15555	14141	9628	0.33	0.19	0.12	0.1	0.07
5	22.78	25.56	25.44	25.66	25.82	31911	17052	11996	10046	7056	0.21	0.09	0.04	0.04	0.05
	Panel J. Market Leverage bottom 30th pctl					Panel K. Book Leverage top 30th pctl					Panel L. Market Leverage top 30th pctl				
1	0.8	0.65	0.51	0.38	0.21	0.19	0.11	0.11	0.1	0.13	0.03	0.04	0.06	0.1	0.28
2	0.85	0.67	0.43	0.27	0.13	0.13	0.09	0.09	0.11	0.17	0.01	0.03	0.04	0.09	0.3
3	0.85	0.6	0.35	0.2	0.1	0.13	0.1	0.12	0.13	0.15	0.02	0.03	0.06	0.12	0.3
4	0.81	0.57	0.27	0.14	0.06	0.16	0.11	0.13	0.15	0.15	0.01	0.03	0.07	0.15	0.34
5	0.8	0.4	0.17	0.07	0.06	0.17	0.15	0.19	0.2	0.14	0.02	0.05	0.12	0.22	0.26

Table 6: Size and Book-to-Market Portfolio Returns and the Small Growth Anomaly.

The table reports estimated intercepts for the return regressions on the Fama and French 3-factors. At the end of each June, we rank and sort stocks into 5 groups based on market equity (size) and, separately, into 5 groups based on book-to-market ratio where the cutoff values are based on NYSE firms, then 25 5×5 monthly value-weighted returns are computed for each of the 25 portfolios. The regression model is

$$r_t - r_{f,t} = \alpha + \gamma_1 SMB_t + \gamma_2 HML_t + \gamma_3 MKTRF_t + \epsilon_t$$

where r_t is portfolio return, $r_{f,t}$ is the monthly riskless rate, SMB, HML and $MKTRF$ are the Fama and French (1993) three factors that proxy for size, book-to-market and the market risk premium respectively. Estimates are also reported for the zero-cost portfolios (column and row labeled 5-1) and the portfolios equally-weighed the portfolios along each one-way rank classification of size and book-to-market (column and row labeled Mean). All portfolios are rebalanced monthly and the reported intercepts are annualized. Newey and West (1987) robust t-statistics are reported in square brackets.

	book-to-market						
size	1	2	3	4	5	5-1	Mean
1	-5.5567***	3.1782**	2.7962**	3.4110***	2.4822**	8.0389***	1.2622
	[-3.2479]	[1.9997]	[2.4420]	[3.5441]	[2.1203]	[4.2272]	[1.3012]
2	-1.6737	0.0837	1.7327	1.2034	-1.7025	-0.0288	-0.0713
	[-1.5453]	[0.0707]	[1.4832]	[1.2329]	[-1.2962]	[-0.0203]	[-0.0937]
3	0.0291	1.7887	-0.7231	0.0182	0.4238	0.3946	0.3073
	[0.0255]	[1.1764]	[-0.5572]	[0.0144]	[0.2996]	[0.2064]	[0.3603]
4	2.1421*	0.8103	-0.567	-1.1286	0.2589	-1.8833	0.3031
	[1.9398]	[0.6509]	[-0.4331]	[-0.8270]	[0.1769]	[-1.0963]	[0.3495]
5	2.4107***	0.3107	0.3694	-1.9892	-1.0861	-3.4968*	0.0031
	[2.9061]	[0.2635]	[0.3309]	[-1.5386]	[-0.6210]	[-1.7693]	[0.0060]
5-1	7.9675***	-2.8675	-2.4268	-5.4002***	-3.5683*		-1.2591
	[4.2499]	[-1.3727]	[-1.6374]	[-3.6956]	[-1.6524]		[-1.2259]
Mean	-0.5297	1.2343	0.7216	0.3029	0.0752	0.6049	
	[-0.7270]	[1.5196]	[0.9741]	[0.3927]	[0.0909]	[0.6139]	

Table 7: O-Score Portfolio Returns and the Distress Anomaly.

Panel A of the table reports the mean returns and the estimated return intercepts from the CAPM, the Fama and French 3-factor model, and the Cahart 4-factor regressions for each of the O-Score decile portfolios. At the end of each June, we sort stocks evenly into 10 groups based on O-Scores, then we compute monthly value-weighted returns for portfolio. Estimated intercepts are also reported for the zero-cost portfolio that is long and short the top and the bottom decile portfolios (column and row labeled 10-1). Panels B and C of the table report intercept estimates for each book leverage and market leverage tercile after stocks are sorted and ranked based on O-Score and, independently, based on book leverage and market leverage ratios. All portfolios are rebalanced monthly and the reported estimates are annualized. Newey and West (1987) robust t-statistics are reported in square brackets.

	1	2	3	4	5	6	7	8	9	10	10-1
Panel A. O-Score decile portfolio returns											
mean return	12.42***	11.01***	12.84***	12.52***	12.09***	12.38***	11.78***	13.98***	09.41*	3.78	-8.64
	[3.0904]	[3.7510]	[4.6256]	[4.3607]	[4.1254]	[4.1372]	[3.5640]	[3.5445]	[1.9220]	[0.5780]	[-2.0432]
CAPM alpha	0.2399	0.0996	2.1387*	1.6160*	1.416	1.6550*	0.5514	1.6005	-3.8885*	-11.1285***	-11.3683***
	[0.1135]	[0.0979]	[1.9143]	[1.6803]	[1.3061]	[1.7042]	[0.3874]	[1.0062]	[-1.7107]	[-2.7605]	[-2.9804]
3-factor alpha	3.7398**	0.9269	1.8172*	0.9188	-0.123	0.4756	-1.309	0.55	-3.8098**	-9.2787***	-13.0185***
	[2.4221]	[1.0014]	[1.9463]	[1.0132]	[-0.1249]	[0.4929]	[-1.0694]	[0.4542]	[-2.0471]	[-3.3657]	[-4.5889]
4-factor alpha	3.4432**	1.2841	1.1455	1.1795	-0.3759	-0.2977	-1.5021	0.8944	-2.9298	-8.0603***	-11.5034***
	[2.3714]	[1.2540]	[1.1666]	[1.2592]	[-0.3913]	[-0.2870]	[-1.1099]	[0.7441]	[-1.6141]	[-2.8670]	[-3.8948]
Panel B. O-Score decile portfolio 3-factor alphas for each book leverage tercile											
1	2.6493	0.4159	-0.8699	3.6531	-3.8125	0.045	-7.0062*	-0.5426	-3.2749	-9.2617**	-11.9110***
	[1.5258]	[0.1803]	[-0.3316]	[0.9506]	[-1.1679]	[0.0140]	[-1.7641]	[-0.1345]	[-0.6712]	[-2.3861]	[-3.0270]
2	4.2821**	1.5807	1.3222	0.2078	-0.5763	-1.6321	-1.0916	-4.1147	-1.1378	-11.7023***	-15.9844***
	[1.9999]	[1.4749]	[1.2160]	[0.1719]	[-0.3836]	[-1.0105]	[-0.5630]	[-1.5032]	[-0.2635]	[-2.8530]	[-3.4889]
3	0.4745	2.3486	3.2594*	1.4638	0.7265	1.1521	-0.5237	1.0782	-5.4104**	-6.6141*	-10.676
	[0.0370]	[0.4937]	[1.8199]	[1.0983]	[0.5713]	[0.8750]	[-0.3815]	[0.7844]	[-2.4228]	[-1.9350]	[-0.8412]
Panel C. O-Score decile portfolio 3-factor alphas for each market leverage tercile											
1	3.6390**	0.384	3.1042*	-1.4272	-0.3117	3.7269	-7.0831**	-7.5777**	-9.8926**	-12.2324***	-15.8714***
	[2.1971]	[0.3447]	[1.7694]	[-0.7203]	[-0.1057]	[1.0805]	[-2.0105]	[-2.1851]	[-2.5548]	[-3.7294]	[-4.7695]
2	-0.9916	1.2201	1.3089	0.8293	0.4848	-0.6787	-3.5846*	-0.8333	-5.9926**	-7.028	-6.0364
	[-0.2312]	[0.8873]	[1.0615]	[0.7516]	[0.3934]	[-0.4615]	[-1.9138]	[-0.4261]	[-2.0636]	[-1.5657]	[-0.8702]
3	18.0339	-4.7896	1.6459	3.5516**	-0.3217	0.7861	-0.6353	2.1053	-2.5294	-1.5328	-13.4858
	[1.4047]	[-0.6835]	[0.7602]	[1.9695]	[-0.2355]	[0.5168]	[-0.4089]	[1.3979]	[-1.1051]	[-0.3293]	[-0.8556]

Table 8: Trading Strategy Return Regressions across Financial Leverage Terciles

Panel A of the table reports the coefficient estimates from the time series regressions of the small growth trading strategy returns on the Fama and French three factors and the distress trading strategy returns. The regression model is

$$SG_t = \alpha + \gamma_1 SMB_t + \gamma_2 HML_t + \gamma_3 MKTRF_t + \gamma_4 DISTRESS_t + \epsilon_t$$

where SMB, HML and $MKTRF$ are the Fama and French (1993) three factors that proxy for size, book-to-market and the market risk premium respectively, and SG and $DISTRESS$ are the trading strategy returns for small growth and distress respectively. At the end of each June, firms are ranked and sorted into five groups based on size and five groups based on book-to-market ratio where the cutoff values are based on NYSE firms, and separately into 10 equally sized groups based on O-Score, then monthly value-weighted portfolio returns are computed for each of the 5×5 size and book-to-market groups, and separately for each O-Score group. SG is the zero-cost portfolio return that is long the lowest size and the lowest book-to-market portfolio and short the middle size and the middle book-to-market portfolio. $DISTRESS$ is the zero-cost portfolio that is long and short the top and the bottom O-Score decile portfolios. Panels B and C of the table report the trading strategy regression estimates for each book leverage and market leverage tercile after stocks are sorted and ranked based on size, book-to-market, O-Score, book leverage, market leverage. All portfolios in the trading strategies are rebalanced monthly. Column $\alpha \times 12$ corresponds to annualized intercept estimates. Newey and West (1987) robust t-stats are reported in square brackets.

	α	$\alpha \times 12$	SMB	HML	$MKTRF$	$DISTRESS$	RSq	Adj. RSq
				Panel A. Full Sample				
	0.0589	0.7065	0.3031***	-0.8740***	-0.0194	0.4256***	0.7078	0.7044
	[0.2595]	[0.2595]	[2.8080]	[-7.2750]	[-0.3952]	[7.6875]		
				Panel B. Book Financial Leverage				
1	-0.0235	-0.2815	0.3790***	-0.6701***	0.0353	0.2697***	0.4751	0.4691
	[-0.0884]	[-0.0884]	[2.8099]	[-4.0743]	[0.5063]	[6.1678]		
2	-0.2216	-2.6591	0.6501***	-0.8052***	0.0474	0.1660***	0.6063	0.6018
	[-1.0079]	[-1.0079]	[3.9399]	[-6.8863]	[0.9542]	[4.7208]		
3	-0.3925	-4.7101	0.6337***	-0.3910***	0.0895	-0.0166	0.2479	0.2313
	[-1.3401]	[-1.3401]	[4.6182]	[-3.0670]	[1.4762]	[-1.0199]		
				Panel C. Market Financial Leverage				
1	-0.1143	-1.371	0.1944	-0.4837***	-0.0273	0.3044***	0.2525	0.2439
	[-0.3336]	[-0.3336]	[1.2633]	[-3.4660]	[-0.3458]	[3.8348]		
2	0.0371	0.4448	0.7813***	-0.7136***	0.0216	0.1302*	0.5112	0.5056
	[0.1562]	[0.1562]	[4.7088]	[-4.1657]	[0.3758]	[1.7989]		
3	-0.1672	-2.0059	0.7092***	-0.2367	0.1457	-0.0043	0.1525	0.1316
	[-0.3304]	[-0.3304]	[3.0892]	[-0.8385]	[1.2211]	[-0.0934]		

Table 9: Trading Strategy Return Regressions across Growth Option Intensity Terciles

This table reports the coefficient estimates from the time series regressions of the small growth trading strategy returns on the Fama and French three factors and the distress trading strategy returns for each growth option tercile. The regression model is

$$SG_t = \alpha + \gamma_1 SMB_t + \gamma_2 HML_t + \gamma_3 MKTRF_t + \gamma_4 DISTRESS_t + \epsilon_t$$

where SMB, HML and $MKTRF$ are the Fama and French (1993) three factors that proxy for size, book-to-market and the market risk premium respectively, and SG and $DISTRESS$ are the trading strategy returns for small growth and distress respectively. At the end of each June, firms are ranked and sorted into five groups based on size, and into five groups based on book-to-market where the cutoff values are based on NYSE firms, and separately into 10 equally sized groups based on O-Score and into 3 equally sized groups based on growth option intensity, then monthly value-weighted portfolio returns are computed for each group in each growth option tercile. SG is the zero-cost portfolio return that is long the lowest size and the lowest book-to-market portfolio and short the middle size and the middle book-to-market portfolio. $DISTRESS$ is the zero-cost portfolio that is long and short the top and the bottom O-Score decile portfolios. Panel A of the table reports the estimates when age proxies for growth option intensity. Panels B, C and D report estimates when size (total assets), future sales growth and R&D investments proxy for growth option intensity. All portfolios in the trading strategies are rebalanced monthly. Column $\alpha \times 12$ corresponds to annualized intercept estimates. Newey and West (1987) robust t-stats are reported in square brackets.

	α	$\alpha \times 12$	SMB	HML	$MKTRF$	$DISTRESS$	RSq	Adj. RSq
				Panel A. Age				
1	0.1154	1.3853	0.4200***	-0.8987***	-0.0131	0.3502***	0.5899	0.5852
	[0.5592]	[0.5592]	[3.2364]	[-7.0262]	[-0.2989]	[7.4271]		
2	-0.0536	-0.6433	0.5685***	-0.6948***	-0.0712	0.2434***	0.4728	0.4665
	[-0.1965]	[-0.1965]	[4.7527]	[-4.9654]	[-1.2112]	[6.1639]		
3	-0.3791	-4.5496	0.7821***	-0.6540**	0.0464	0.0999**	0.246	0.2304
	[-0.5076]	[-0.5076]	[3.4108]	[-2.1126]	[0.2221]	[2.0260]		
				Panel B. Size				
1	-0.1363	-1.6357	0.3874***	-0.6004***	-0.0453	0.3776***	0.3059	0.2977
	[-0.4751]	[-0.4751]	[3.2263]	[-3.5510]	[-0.6897]	[5.5575]		
2	0.3962	4.7545	0.3678	-0.3517	0.4983**	0.1543*	0.0622	0.0491
	[0.3539]	[0.3539]	[1.0956]	[-0.6764]	[2.3514]	[1.7022]		
3	6.6430*	79.7159*	1.3433	2.1553**	-0.3633	0.1487	0.208	-0.056
	[1.8780]	[1.8780]	[0.5565]	[2.2180]	[-0.5844]	[0.5482]		
				Panel C. Sales Growth				
1	-0.1964	-2.3573	0.6637***	-0.9317***	-0.0586	0.2020***	0.5311	0.5255
	[-0.7568]	[-0.7568]	[5.2757]	[-5.9308]	[-1.0380]	[4.3026]		
2	-0.2457	-2.9481	0.6978***	-0.8261***	0.0175	0.1518***	0.5358	0.5301
	[-0.9032]	[-0.9032]	[5.1173]	[-5.1051]	[0.2505]	[4.2387]		
3	0.2926	3.5116	0.4353***	-0.7855***	0.0004	0.3294***	0.612	0.6073
	[1.2273]	[1.2273]	[2.9495]	[-6.7016]	[0.0066]	[6.8256]		
				Panel D. R&D				
1	-0.7108**	-8.5299**	0.3474**	-0.7715***	0.0493	0.2553***	0.3433	0.3358
	[-2.0782]	[-2.0782]	[2.1192]	[-5.2753]	[0.5640]	[4.0359]		
2	0.2646	3.1755	0.6049***	-0.6010***	0.0231	0.2016***	0.3515	0.3441
	[0.7800]	[0.7800]	[2.7957]	[-3.2462]	[0.2130]	[4.6880]		
3	0.2066	2.4798	0.6571***	-0.8616***	-0.033	0.2729***	0.4321	0.4256
	[0.6822]	[0.6822]	[3.5794]	[-5.2351]	[-0.4605]	[4.0906]		